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ON APPROXIMATE SOLUTIONS OF FRACTIONAL ORDER SMOKING EPIDEMIC MODEL USING SUMUDU DECOMPOSITION METHOD

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Abstract: In this paper, an approximate analytical solution of a fractional-order smoking epidemic model is obtained by employing the Sumudu Decomposition Method (SDM). To validate the effectiveness and reliability of the proposed method, the obtained SDM solutions are compared with numerical solutions computed via the classical Runge–Kutta method. A detailed error and convergence analysis is presented to assess the accuracy of the method. The results demonstrate that the SDM provides highly accurate approximations, particularly for small and moderate time intervals. Graphical simulations are included to illustrate the dynamical behavior of the model for different parameter values. The study confirms that the SDM is a simple, efficient, and reliable technique for solving nonlinear fractional-order epidemic models and can be effectively applied to analyze short-term dynamics in smoking-related population systems.

Keywords and Phrases: Caputo fractional derivative, Sumudu Decomposition Method, Sumudu Transform, Adomian Polynomials.

2020 Mathematics Subject Classification: 34A08, 92D30.

1. Introduction

In this paper, a mathematical model of smoking epidemic is considered in Caputo fractional derivative sense. Smoking is one of the major reasons for lung cancer. It is also responsible for many other health related problems in human bodies. Unlike other epidemics which can be controlled with proper vaccination, smoking on the other hand is a completely different scenario. Nowadays, smoking can be considered as an epidemic because it has become very popular among the youths going to schools and colleges which not only deteriorates their health but also have a big impact on the development of a society and nation. Many researchers have been working in this field. Our work is to give an approximate solution to the smoking epidemic model. The Sumudu Decomposition Method (SDM) is used to carry out this task. The benefit of taking fractional order lies in its ability to capture memory effects which gives more accurate approach to understand real world problems which is not possible with classical integer order derivatives.

Sumudu Transform Method (STM) and properties were discuused by Belgacem et .al. [4]. Adomian Decomposition Method (ADM) was discussed by Adomian in his paper [1] where he gave the notion of Adomian polynomials. Sumudu Decomposition Method (SDM) which is a combination of STM and ADM was used by Mahdy et. al. [7] to solve fractional Riccati equation. Many other researchers like Baleanu et. al [5] also discussed the modelling of epidemic childhood diseases using Laplace Decomposition Method. Lestari et. al. [6] studied about the smoking behaviour model. Several recent studies have focused on the development of efficient computational techniques for solving fractional and mixed integro-differential equations, including least-squares methods, polynomial-based approaches, and numerical schemes for problems with singular kernels and variable coefficients [2, 3, 8, 9, 10, 11, 12, 13, 14, 16]. These contributions highlight the importance of developing robust and accurate methods for handling complex fractional models. Thus, the main objective of this paper is to obtain approximate solutions of a fractional-order smoking epidemic model using the SDM and to validate its effectiveness through comparison with numerical solutions and error analysis.

The paper has been organized as follows: The first section gives an introduction about the work. The second section is about the preliminaries and definitions to be used. The third section presents methodology of Sumudu Decomposition Method (SDM). The fourth section consists of model formulation. The fifth section contains the approximate solution of the problem. The sixth section illustrates the results in the form of graphs along with discussions on error and convergence analysis. The final section presents the conclusion and future scope of the paper.

2. Preliminaries and Definitions

Definition 2.1. [18] *The Riemann–Liouville fractional integral of order* $\phi \in \mathbb{R}_+$

for $t > 0$ is defined as

$$I_{0,t}^{\phi} f(t) = \frac{1}{\Gamma(\phi)} \int_0^t (t-\xi)^{\phi-1} f(\xi) d\xi \tag{2.1}$$

where $\Gamma(\phi)$ represents the gamma function of ϕ.

Definition 2.2. [17] *The Caputo fractional derivative of order $\phi \in \mathbb{R}_+$ for $t > 0$ is defined as*

$${}^{c}D_{0,t}^{\phi} f(t) = I^{n-\phi}\left({}^{c}D_{0,t}^{\phi}\right) f(t) = \frac{1}{\Gamma(n-\phi)} \int_0^t (t-\xi)^{n-\phi-1} f^{(n)}(\xi) d\xi \tag{2.2}$$

where $n = [\phi] + 1$, with $[\phi]$ being the largest integer less than or equal to ϕ.

Definition 2.3. [4] *The Sumudu transform is defined over the set of functions*

$$A = \{ f(t) : \ \exists\, K, r_1, r_2 > 0, \ |f(t)| < K e^{|t|/r_j}, \ if \ \ t \in (-1)^j \times [0, \infty) \ \}$$

using the following formula:

$$F(u) = S[f(t)] = \frac{1}{u} \int_0^{\infty} e^{-t/u} f(t) f t, \ \ u \in (-r_1, r_2) \tag{2.3}$$

where S is the Sumudu transform operator.

Some properties of the Sumudu transform as given in [4] are:

1. $S\{1\} = 1$
2. $S\{t^n\} = u^n \Gamma(n+1), \ \ n > 0$
3. $S\{f(t) \pm g(t)\} = S\{f(t)\} \pm S\{g(t)\}$

Definition 2.4. [7] *The Sumudu transform of Caputo fractional derivative is defined as*

$$S\{{}^{c}D_{0,t}^{\phi} f(t)\} = u^{-\phi} S\{f(t)\} - \sum_{k=0}^{n-1} u^{-\phi+k} f^{(k)}(0), \ \ n = [\phi] + 1 \tag{2.4}$$

3. Methodology for SDM

Let us consider a system of fractional differential equation

$${}^{c}D_{0,t}^{\phi} f_i(t) = F_i(f_1, f_2, \ldots, f_m, t), \ i = 1, 2, \ldots, m \tag{3.1}$$

with initial conditions

$$f_i^{(k)}(0) = f_{i_0}^k \ , \ i = 1, 2, \ldots, m, \ 0 \leq k \leq [\phi] \tag{3.2}$$

where ${}^cD_{0,t}^{\phi}$ is the Caputo derivative of order ϕ where $n-1 \leq \phi < n$, n being a positive integer, F_i is a linear/non-linear functions.
We first apply Sumudu transform on both sides of Eq.(3.1) and obtain

$$S\{{}^cD_{0,t}^{\phi} f_i(t)\} = S\{F_i\}$$

Using Eq. (2.4) and initial condition (3.2), we have

$$S\{f_i(t)\}u^{-\phi} - \sum_{k=0}^{n-1} u^{-\phi+k} f_i^{(k)}(0) = S\{R(f_i)\} + S\{N(f_i)\}$$

$$or, \ \ S\{f_i(t)\} = \sum_{k=0}^{n-1} u^k f_i^{(k)}(0) + u^{\phi} S\{R(f_i)\} + u^{\phi} S\{N(f_i)\} \tag{3.3}$$

where R is linear bounded operator and N is a nonlinear bounded operator.

The SDM gives the solution $f_i(t)$ in series form as

$$f_i(t) = \sum_{n=0}^{\infty} f_{i_n}(t) \tag{3.4}$$

and the nonlinear term $N(f_i)$ takes the form

$$N(f_i) = \sum_{n=0}^{\infty} A_n \tag{3.5}$$

where A_n is the Adomian polynomials of $f_{i_0}, f_{i_1}, f_{i_2}, \ldots, f_{i_n}$ and it is given by

$$A_n = \frac{1}{\Gamma(n+1)} \frac{d^n}{d\lambda^n} \left[N\left(\sum_{n=0}^{\infty} \lambda^n f_{i_n} \right) \right]_{\lambda=0} \ , \ n = 0, 1, 2, \ldots \tag{3.6}$$

Using Eq. (3.4) and (3.5) in Eq. (3.3), we get

$$S\left\{ \sum_{n=0}^{\infty} f_{i_n}(t) \right\} = \sum_{k=0}^{n-1} u^k f_i^{(k)}(0) + u^{\phi} S\left\{ R\left(\sum_{n=0}^{\infty} f_{i_n}(t) \right) \right\} + u^{\phi} S\left\{ \sum_{n=0}^{\infty} A_n \right\} \tag{3.7}$$

Comparing both sides of above Eq. (3.7), we get

$$S\{f_{i_0}\} = \sum_{k=0}^{n-1} u^k f_i^{(k)}(0) \tag{3.8}$$

$$S\{f_{i_1}\} = u^{-\phi} S\{R(f_{i_0})\} + u^{\phi} S\{A_0\} \tag{3.9}$$

$$S\{f_{i_2}\} = u^{-\phi} S\{R(f_{i_1})\} + u^{\phi} S\{A_1\} \tag{3.10}$$

We get a recursive relation in general as follows

$$S\{f_{i_n}\} = u^{-\phi} S\{R(f_{i_{n-1}})\} + u^{\phi} S\{A_{n-1}\}\ ,\ n = 1, 2, 3, \ldots \tag{3.11}$$

Applying inverse Sumudu transform from Eq. (3.8) to (3.11), we get

$$f_{i_0} = G(t) \tag{3.12}$$

$$f_{i_n} = S^{-1}\{u^{-\phi} S\{R(f_{i_{n-1}})\} + u^{\phi} S\{A_{n-1}\}\}\ ,\ n = 1, 2, 3, \ldots \tag{3.13}$$

where $G(t)$ is a function that comes from the source term and the given initial conditions.

4. Formulation of the Model

A mathematical model on smoking epidemic model was given by Lestari [6] as follows

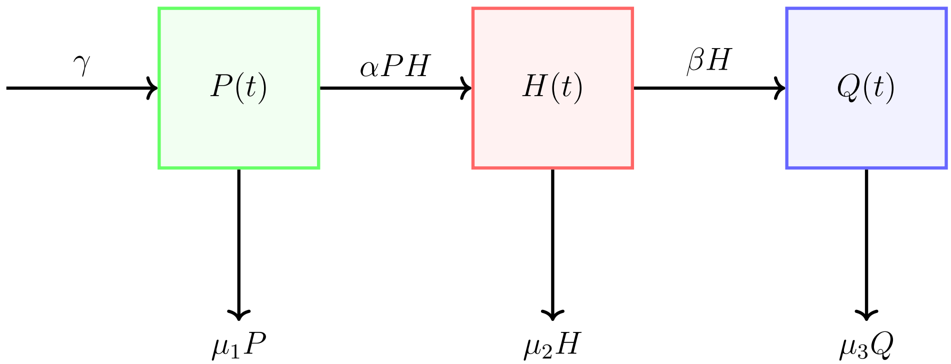
$$\begin{aligned} \frac{dP}{dt} &= \gamma - \alpha PH - \mu_1 P \\ \frac{dH}{dt} &= \alpha PH - \beta H - \mu_2 H \\ \frac{dQ}{dt} &= \beta H - \mu_3 Q \end{aligned} \tag{4.1}$$

where $P(t), H(t), Q(t)$ denote the populations of potential smokers, heavy smokers, and quit smokers, respectively, γ represents the recruitment rate, which corresponds to the natural birth rate, α signifies the transmission rate of smoking habits, influenced by the interaction between individuals who have never smoked and heavy smokers, β denotes the rate at which smokers quit smoking, μ_1 represents the natural death rate of the potential smoker population, μ_2 corresponds to the death rate of the heavy smoker population, and μ_3 indicates the death rate of individuals

who have quit smoking, with $\mu_1 \leq min\{\mu_2, \mu_3\}$.
We introduce the fractional multi-order model as follows

$$\begin{aligned} {}^{c}D_{0,t}^{\phi_1}P &= \gamma - \alpha PH - \mu_1 P \\ {}^{c}D_{0,t}^{\phi_2}H &= \alpha PH - \beta H - \mu_2 H \\ {}^{c}D_{0,t}^{\phi_3}Q &= \beta H - \mu_3 Q \end{aligned} \tag{4.2}$$

with initial conditions $P(0) = 100$, $H(0) = 40$ and $Q(0) = 10$, where $0 < \phi_1,\ \phi_2,\ \phi_3 \leq 1$ are the fractional-order parameters.



5. Approximate solution of the problem by SDM

Applying SDM on both sides of Eq. (4.2), we get

$$\begin{aligned} S\{{}^{c}D_{0,t}^{\phi_1}P\} &= S\{\gamma\} - S\{\alpha PH\} - S\{\mu_1 P\} \\ S\{{}^{c}D_{0,t}^{\phi_2}H\} &= S\{\alpha PH\} - S\{\beta H\} - S\{\mu_2 H\} \\ S\{{}^{c}D_{0,t}^{\phi_3}Q\} &= S\{\beta H\} - S\{\mu_3 Q\} \end{aligned} \tag{5.1}$$

which gives us

$$\begin{aligned} S\{P(t)\} &= P(0) + u^{\phi_1}S\{\gamma\} - u^{\phi_1}\alpha S\{PH\} - u^{\phi_1}\mu_1 S\{P\} \\ S\{H(t)\} &= H(0) + u^{\phi_2}\alpha S\{PH\} - u^{\phi_2}\beta S\{H\} - u^{\phi_2}\mu_2 S\{H\} \\ S\{Q(t)\} &= Q(0) + \beta S\{H\} - \mu_3 S\{Q\} \end{aligned} \tag{5.2}$$

Applying inverse Sumudu transform on (5.2) and decomposition method, we obtain a recursive formula

$$\begin{aligned} P_0(t) &= P(0) + \frac{\gamma t^{\phi_1}}{\Gamma(\phi_1 + 1)} \\ P_{n+1}(t) &= -S^{-1}[\alpha u^{\phi_1}S\{A_n\} + \mu_1 u^{\phi_1}S\{P_n(t)\}] \end{aligned} \tag{5.3}$$

and

$$\begin{aligned} H_0(t) &= H(0) \\ H_{n+1}(t) &= S^{-1}[\alpha u^{\phi_2} S\{A_n\} - u^{\phi_2}(\beta+\mu_2) S\{H_n(t)\}] \end{aligned} \tag{5.4}$$

and

$$\begin{aligned} Q_0(t) &= Q(0) \\ Q_{n+1}(t) &= S^{-1}[\beta u^{\phi_3} S\{H_n\} - \mu_3 u^{\phi_3} S\{Q_n(t)\}] \end{aligned} \tag{5.5}$$

where A_n are Adomian polynomials, which are calculated as follows [5]:

$$A_n(t) = \frac{1}{\Gamma(n+1)} \frac{d^n}{d\lambda^n} \left[\sum_{k=0}^{n} \lambda^k P_k(t) \sum_{k=0}^{n} \lambda^k H_k(t) \right]_{\lambda=0} \tag{5.6}$$

We can get the first three terms as $A_0 = P_0(t)H_0(t)$, $A_1 = P_0(t)H_1(t) + P_1(t)H_0(t)$ and $A_2 = 2P_0(t)H_2(t) + 2P_1(t)H_1(t) + 2P_2(t)H_0(t)$.
Thus, we can obtain the following successive approximations

$$\begin{aligned} P_1(t) &= \frac{t^{2\phi_1}(-40\alpha\gamma - \gamma\mu_1)}{\Gamma(2\phi_1+1)} + \frac{(-4000\alpha - 100\mu_1)t^{\phi_1}}{\Gamma(\phi_1+1)} \\ H_1(t) &= \frac{40\alpha\gamma t^{\phi_1+\phi_2}}{\Gamma(\phi_1+\phi_2+1)} + \frac{t^{\phi_2}(4000\alpha - 40(\beta+\mu_2))}{\Gamma(\phi_2+1)} \\ Q_1(t) &= \frac{(40\beta - 10\mu_3)t^{\phi_3}}{\Gamma(\phi_3+1)} \\ P_2(t) &= -\frac{40\alpha^2\gamma^2 t^{3\phi_1+\phi_2}\Gamma(2\phi_1+\phi_2+1)}{\Gamma(\phi_1+1)\Gamma(\phi_1+\phi_2+1)\Gamma(3\phi_1+\phi_2+1)} - \\ &\quad \frac{\alpha t^{2\phi_1+\phi_2}\Gamma(\phi_1+\phi_2+1)\left(\frac{\gamma(4000\alpha-40(\beta+\mu_2))}{\Gamma(\phi_1+1)\Gamma(\phi_2+1)} + \frac{4000\alpha\gamma}{\Gamma(\phi_1+\phi_2+1)}\right)}{\Gamma(2\phi_1+\phi_2+1)} - \\ &\quad \frac{100\alpha(4000\alpha - 40(\beta+\mu_2))t^{\phi_1+\phi_2}}{\Gamma(\phi_1+\phi_2+1)} - \frac{40\alpha t^{3\phi_1}(-40\alpha\gamma - \gamma\mu_1)}{\Gamma(3\phi_1+1)} - \\ &\quad \frac{40\alpha(-4000\alpha - 100\mu_1)t^{2\phi_1}}{\Gamma(2\phi_1+1)} \end{aligned}$$

$$H_2(t) = \frac{40\alpha^2\gamma^2 t^{2\phi_1+2\phi_2}\Gamma(2\phi_1+\phi_2+1)}{\Gamma(\phi_1+1)\Gamma(\phi_1+\phi_2+1)\Gamma(2\phi_1+2\phi_2+1)} +$$

$$\frac{t^{\phi_1+2\phi_2}\left(\alpha\Gamma(\phi_1+\phi_2+1)\left(\frac{\gamma(4000\alpha-40(\beta+\mu_2))}{\Gamma(\phi_1+1)\Gamma(\phi_2+1)} + \frac{4000\alpha\gamma}{\Gamma(\phi_1+\phi_2+1)}\right) - 40\alpha\gamma(\beta+\mu_2)\right)}{\Gamma(\phi_1+2\phi_2+1)} +$$

$$\frac{t^{2\phi_1}(100\alpha(4000\alpha-40(\beta+\mu_2)) - (\beta+\mu_2)(4000\alpha-40(\beta+\mu_2)))}{\Gamma(2\phi_1+1)} +$$

$$\frac{40\alpha(-40\alpha\gamma-\gamma\mu_1)t^{2\phi_1+\phi_2}}{\Gamma(2\phi_1+\phi_2+1)} + \frac{40\alpha(-4000\alpha-100\mu_1)t^{\phi_1+\phi_2}}{\Gamma(\phi_1+\phi_2+1)}$$

$$Q_2(t) = \frac{40\alpha\beta\gamma t^{\phi_1+\phi_2+\phi_3}}{\Gamma(\phi_1+\phi_2+\phi_3+1)} + \frac{\beta(4000\alpha-40(\beta+\mu_2 2))t^{\phi_2+\phi_3}}{\Gamma(\phi_2+\phi_3+1)} - \frac{\mu_3(40\beta-10\mu_3)t^{2\phi_3}}{\Gamma(2\phi_3+1)}$$

and so on.

6. Results and Discussions

In this section, we discuss the results with the help of graphs with parameter values taken from [6]. Fig. 1 and Fig. 2 represents the effect of rate of transmission of smoking habits parameter α on potential smoker $P(t)$ and heavy smoker $H(t)$. We can observe that owing to an increase in α, the potential smoker $P(t)$ decreases, whereas heavy smoker $H(t)$ increases. Fig. 3 and Fig. 4 highlights the effect of rate of smokers quitting smoking parameter β on heavy smoker $H(t)$ and quit smoker $Q(t)$. It shows that an increase in β decreases heavy smokers $H(t)$ but increases quit smokers $Q(t)$. Fig. 5, Fig. 6 and Fig. 7 shows the variation of $P(t), H(t)$ and $Q(t)$ for different values of fractional order parameters ϕ_1, ϕ_2 and ϕ_3 respectively, illustrating that the fractional orders provide additional degrees of freedom to capture memory effects and significantly influence the system dynamics.

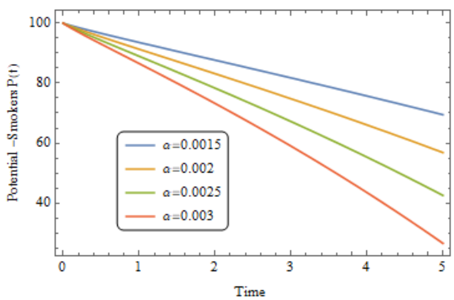


Figure 1: P(t) vs t (α varies)

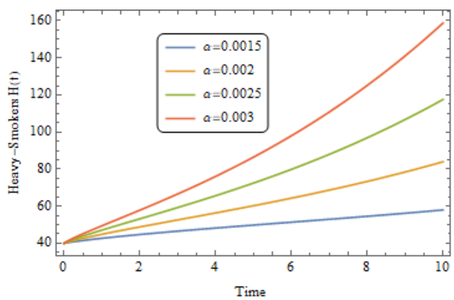


Figure 2: H(t) vs t (α varies)

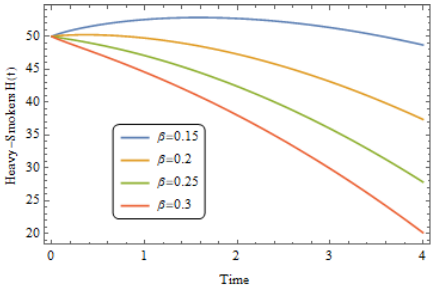


Figure 3: H(t) vs t (β varies)

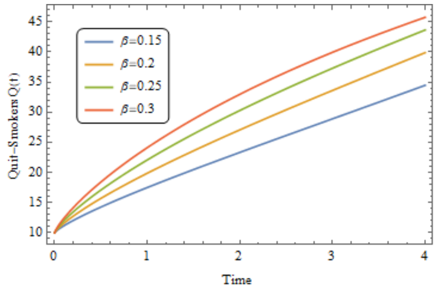


Figure 4: Q(t) vs t (β varies)

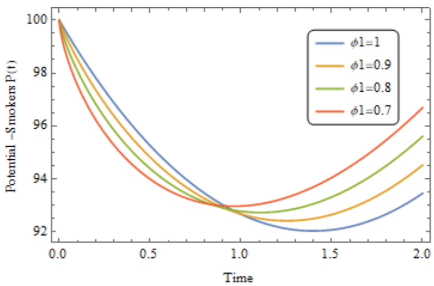


Figure 5: P(t) vs t (ϕ_1 varies)

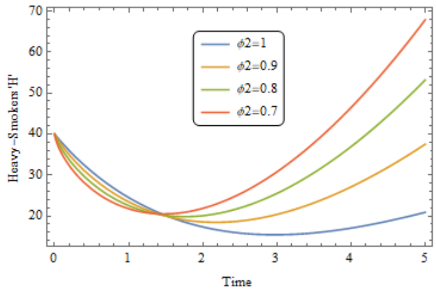


Figure 6: H(t) vs t (ϕ_2 varies)

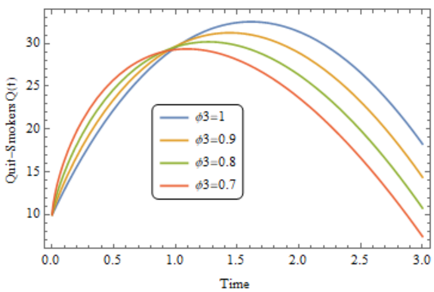


Figure 7: Q(t) vs t (ϕ_3 varies)

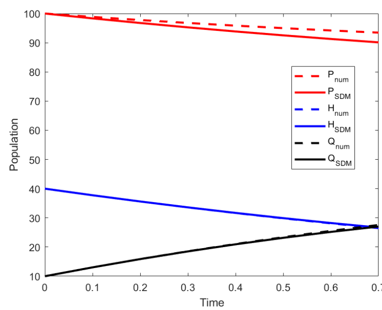


Figure 8: Error Analysis

6.1. Error and Convergence Analysis

We examine the accuracy and computational efficiency of the Sumudu Decomposition Method (SDM) by comparing the truncated series solution with the numerical solution obtained via the classical fourth-order Runge–Kutta method (we have used `ode45` in MATLAB). To quantify the accuracy, we compute the absolute error defined as

$$\text{Absolute Error} = |X_{\text{SDM}}(t) - X_{\text{num}}(t)|, \quad X \in \{P, H, Q\}.$$

The absolute errors are evaluated at the selected time points and are presented in Table 1.

Table 1: Absolute error between SDM approximation and numerical solution

Time (t)	Absolute Error in $P(t)$	Absolute Error in $H(t)$	Absolute Error in $Q(t)$
0.1	0.5031	0.0007	0.0025
0.2	1.0072	0.0045	0.0147
0.3	1.5067	0.01409	0.0453
0.4	1.9893	0.0373	0.1167
0.5	2.4368	0.0863	0.2628
0.6	2.8842	0.1353	0.40901
0.7	3.3317	0.1843	0.5551

The error analysis reveals that the absolute error increases with time, indicating that the Sumudu Decomposition Method (SDM) provides a highly accurate approximation in the early stages of the system dynamics. The comparison shows strong agreement between the two approaches, particularly for small and moderate time intervals, confirming the accuracy and reliability of the SDM approximation which is represented in Fig 8.

Note that the SDM solution is obtained in a truncated series form; hence, the observed increase in the relative error for larger time values is a consequence of neglecting higher-order terms. This indicates that the method provides a local approximation around the initial time. Nevertheless, the accuracy can be systematically improved by incorporating additional terms in the series expansion. Therefore, the SDM remains a reliable and computationally efficient semi-analytical technique for analyzing the short-term dynamics of the model.

The convergence of the SDM series can be justified using the theoretical framework of the Adomian decomposition method. In particular, the nonlinear terms in the present model are polynomial in nature and hence infinitely differentiable in the domain of interest. Moreover, the model parameters are bounded, and the state variables remain finite owing to biological constraints. Therefore, the nonlinear operator satisfies the smoothness and boundedness conditions (see Theorems 4.1 and 4.2 in [15]), which ensure the absolute convergence of the decomposition series. Furthermore, the recursive scheme generated by the SDM can be interpreted as a fixed-point iteration. Under the Lipschitz continuity of the nonlinear operator, the sequence of partial sums converges to the unique solution of the system (see Theorems 4.3 and 4.4 in [15]).

This demonstrates that SDM provides a convergent series solution for the proposed smoking model. Moreover, the method ensures high accuracy in only a few terms, making it an efficient and reliable semi-analytical tool for studying the short-term dynamics of fractional-order epidemics.

7. Conclusion

In this study, a fractional multi-order smoking epidemic model formulated in the Caputo sense has been analyzed using the Sumudu Decomposition Method (SDM). The method provides an explicit series-form approximate solution that is simple to implement and avoids complex numerical discretization.

To assess the accuracy and reliability of the proposed approach, the SDM results were compared with numerical solutions obtained using the classical Runge–Kutta method in MATLAB. The comparison demonstrated strong agreement between the two approaches, particularly over small and moderate time intervals. Furthermore, a detailed error analysis confirmed that the SDM yielded highly accurate approximations with minimal computational effort. The convergence analysis showed that the SDM series solution was convergent under suitable smoothness and boundedness conditions on the nonlinear terms.

The numerical simulations illustrate the influence of key parameters on system dynamics. In particular, the fractional-order parameter plays a significant role in governing the memory effect and alters the evolution of the population compartments. The results highlight the capability of fractional models to capture more realistic dynamics than classical integer-order models.

Overall, the findings demonstrate that the SDM is an efficient, reliable, and computationally attractive semi-analytical technique for solving nonlinear fractional-order epidemic models. The method is particularly suitable for short-term analyses and provides valuable insights into the qualitative behavior of the system.

This study can be extended by incorporating optimal control strategies to analyze intervention policies and by considering variable-order fractional derivatives to capture time-dependent memory effects. Further comparisons with other analytical and numerical methods may provide deeper insights into the efficiency of SDM. Additionally, calibrating the model with real-world data and extending it to more complex systems would enhance its practical relevance.

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